ECONOMIC FUNCTIONS IN AN UNCERTAIN ENVIRONMENT

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ABSTRACT

The world is going through one of the deepest economic crises in recent times. The current and future scenarios are riddled with uncertainty that is not always taken into account in the analysis of the different economic functions of interest for the development of the business, and the reality is presented by static and almost perfect models of market behavior.

The fuzzy sets theory allows building suitable models from uncertain realities that present vagueness intrinsically. The use of fuzzy numbers makes possible to consider the aspects of an imprecise environment. Extreme and more possible situations of the different variables involved can be determined through the opinion of experts.

In this paper we present the fuzzy function concept applied to a case study involving economic functions, where the uncertainty is incorporated through the use of triangular fuzzy numbers.

KEY WORDS: Fuzzy Number; Fuzzy Function; Economic Functions.

INTRODUCTION

The world is going through one of the deepest economic crisis in recent times. Both current and future scenarios are fraught with uncertainty.

In general, when is analyzes the various economic functions that are of interest to business development do not take into account the environment of uncertainty, and the reality is represented by static and behavioral nearly perfect market models.

The fuzzy set theory allows building suitable models from uncertain realities that have intrinsically vague; this means that any attempt to exact the used elements leads to a simplification that changes the terms in which problems arise or leads to no real solutions.

The use of fuzzy numbers gives the possibility to consider aspects of an imprecise

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environment. Through expert opinion can be determined more extreme scenarios and different variables in play.

A fuzzy model represents any real system very close way as perceived by individuals; this makes it understandable, even for a non-specialist audience, and each parameter has a meaning easy to understand. This approach shortens the distance between the observer studying the system and creating the mathematical model.

In this paper the concept of function of fuzzy number is presented and, as a novel contribution, it is applied to a case study that involves economic functions, in which the uncertainty is incorporated by using triangular fuzzy numbers.

It is structured as follows: in section 2 the theoretical elements necessary for the study of fuzzy functions are presented; in section 3 concepts of fuzzy number function and its derivative are introduced; and in section 4 these notions are applied to the analysis of a case; and at section 5 some commentary.

DEVELOPMENT

1. Theoretical elements

For a universe E continuous or discrete. A fuzzy subset or fuzzy set $\tilde{A}$ is a function $\mu_{\tilde{A}}: X \rightarrow [0, 1]$ that assigns to each element of the set E a value $\mu_{\tilde{A}}(x)$ that belongs to the range $[0, 1]$, called the degree or level of membership x to $\tilde{A}$ (Zadeh, 1965).

Giving a fuzzy subset $\tilde{A}$ from the referential E, it's called α-cut or level set α from $\tilde{A}$ to the clear set $A_\alpha = \{x \in E / \mu_{\tilde{A}}(x) \geq \alpha\}$ for all $\alpha \in (0, 1]$ (Kaufmann, 1983). This means that, when α-cut of a fuzzy set is the clear set that contains all elements of the reference set whose membership degrees to the fuzzy set is greater than or equal to a specified value α (Lazzari, 2010). If $\alpha = 0$, the α- corresponding cut is the closure of the union of the A_α, with $0 < \alpha \leq 1$ (Buckley y Qu, 1991). The closure of a set A is the smallest closed subset that contain A, ie is the intersection of all closed subsets that contain A, and is denoted A^-. The set A^- is a closed set (Ying-Ming y Mao-Kang, 1997).

A fuzzy set $\tilde{A} \subset E$, is normal if and only if, $\forall\ x \in E,\ \max\ \mu_{\tilde{A}}(x) = 1$ **,** and if convex if and only if, $\forall\ x \in [x_1, x_2] \subset \Re$ verifying that $\mu_{\tilde{A}}(x) \geq \min\ \{\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2)\}$ (Tanaka, 1997).

A fuzzy number (FN) $\tilde{A}$ (Kaufmann y Gupta, 1985; Kaufmann y Gil Aluja, 1990) is a fuzzy subset of real, convex and normal numbers. Fuzzy number can be defined on any totally ordered reference set.

A fuzzy number is continuous if and only if its membership function is a continuous function.

A fuzzy number is positive if its membership function is zero for all real numbers less than or equal to zero: $\tilde{A}$ is positive $\Leftrightarrow \mu_{\tilde{A}}(x)=0 \quad \forall \ x \leq 0$.

The real numbers and intervals of real numbers can be considered as particular cases of fuzzy numbers (Lazzari, 2010).

A fuzzy number can be represented by their α-cut uniquely. Being a convex fuzzy subset, its α-cuts are closed intervals of real numbers. The two ways of expressing a fuzzy number, either by its membership function, $\mu_{\tilde{A}}(x)$, $\forall \ x \in \Re$, or for α-cuts, $A_\alpha = [a_1(\alpha), \ a_2(\alpha)] \ \ \alpha \in [0 \, , 1]$, are equivalents (Kaufmann y Gupta, 1985).

1.1. Extension Principle

This principle was introduced by Zadeh (1975) and is one of the basic ideas of fuzzy set theory, because it allows a fuzzy extension of the classical mathematical concepts such as arithmetic operations.

The first fuzzy set as a subset of a reference may induce other fuzzy reference (or the same) from a given reference function between subset.

If *f* is a function of *X* in *Y* and $\tilde{A}$ a fuzzy subset of the *X* is possible obtain a new fuzzy subset $\tilde{B}=f(\tilde{A})$ of *Y* such that $\tilde{B}=f(\tilde{A})=\{(y, \ \mu_{\tilde{B}}(y)) / \ y=f(x), \ x \in X\}$, where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_{\tilde{A}}(x) & \text{si} \quad f^{-1}(y) \neq \varnothing \\ 0 & \text{si} \quad f^{-1}(y) = \varnothing \end{cases}$$

If f is an inyective function, then $\mu_{\tilde{B}}(y)=\mu_{\tilde{A}}(f^{-1}(y))$ when $f^{-1}(y) \neq \varnothing$.

The importance of this principle lies in the possibility of trying to form any domain fuzzy mathematical reasoning based on the theory of classical ensembles. That is, they can replace a variable which is a value determined by the concept of fuzzy membership degree for each possible value (Ramik, 1986).

1.2. Arithmetic operations with fuzzy numbers

Operations with fuzzy numbers can be performed by using: a) the Extension principle, and b) the α-cuts and the generalization of arithmetic operations with intervals (Buckley et al., 2010; Kaufmann and Gila Aluja, 1990).

Given two real fuzzy numbers $\tilde{A}$ and $\tilde{B}$ by their membership functions $\mu_{\tilde{A}}(x)$ y $\mu_{\tilde{B}}(x)$, when using the extension principle the membership functions for basic arithmetic operations are:

$$\forall x \in \Re : \quad \mu_{\tilde{A}(+)\tilde{B}}(x) = \sup_{x=y+z} \min \left[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(z)\right]$$

$$\mu_{\tilde{A}(-)\tilde{B}}(x) = \sup_{x=y-z} \min \left[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(z)\right]$$

$$\forall x \in \Re^{+} : \quad \mu_{\tilde{A}(.)\tilde{B}}(x) = \sup_{x=y.z} \min \left[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(z)\right]$$

$$\mu_{\tilde{A}(:)\tilde{B}}(x) = \sup_{x=y/z} \min \left[\mu_{\tilde{A}}(y), \mu_{\tilde{B}}(z)\right]$$

Given the α-cuts of a continuous real fuzzy number are close intervals $\Re$, operations between fuzzy numbers can be defined as a generalization of operations between arithmetic intervals (Kaufmann and Gupta, 1985).

Being $\tilde{A}$ and $\tilde{B}$ continuous fuzzy number of $\Re$, express for the α-cuts $A_\alpha=[a_1(\alpha)\ ,\ a_2(\alpha)]$ and $B_\alpha=[b_1(\alpha)\ ,\ b_2(\alpha)]$ for $\alpha\in[0,1]$ (Lazzari, 2010).

- If $\tilde{C}=\tilde{A}(+)\tilde{B}$ then $\tilde{C}_\alpha=A_\alpha(+)B_\alpha$

$$A_\alpha(+)B_\alpha=\left[a_1(\alpha)\ ,\ a_2(\alpha)\right](+)\left[b_1(\alpha)\ ,\ b_2(\alpha)\right]$$
$$A_\alpha(+)\ B_\alpha=\left[a_1(\alpha)+b_1(\alpha), a_2(\alpha)+b_2(\alpha)\right]$$

- If $\tilde{C}=\tilde{A}(-)\tilde{B}$ then $\tilde{C}_\alpha=A_\alpha(-)B_\alpha$

$$A_\alpha(-)B_\alpha=[a_1(\alpha)\ ,\ a_2(\alpha)](-)[b_1(\alpha)\ ,\ b_2(\alpha)]$$
$$A_\alpha(-)B_\alpha=[a_1(\alpha)\ -b_2(\alpha)\ ,\ a_2(\alpha)-b_1(\alpha)]$$

- If $\tilde{C}=\tilde{A}(.)\tilde{B}$ then $\tilde{C}_\alpha=A_\alpha(.)B_\alpha$

$$A_\alpha(.)B_\alpha=[a_1(\alpha)\ ;\ a_2(\alpha)]\ (.)\ \left[b_1(\alpha)\ ;\ b_2(\alpha)\right]$$

$$A_\alpha(.)B_\alpha=[\min\left(a_1(\alpha).b_1(\alpha),\ a_1(\alpha).b_2(\alpha),\ a_2(\alpha).\ b_1(\alpha),\ a_2(\alpha).\ b_2(\alpha)\right);$$
$$\max\left(a_1(\alpha).b_1(\alpha),\ a_1(\alpha).b_2(\alpha),\ a_2(\alpha).\ b_1(\alpha),\ a_2(\alpha).\ b_2(\alpha)\right)]$$

If $\tilde{A}$ and $\tilde{B}$ are continuous fuzzy numbers of $\Re^+$:

$$A_\alpha(.)B_\alpha=[a_1(\alpha)\ .\ b_1(\alpha)\ ;\ a_2(\alpha)\ .\ b_2(\alpha)]$$

- If $\tilde{C}=\tilde{A}(:)\tilde{B}$ then $\tilde{C}_\alpha=A_\alpha(:)B_\alpha$

  If $\tilde{A}$ and $\tilde{B}$ are continuous fuzzy numbers of $\Re^+$:

$$A_\alpha(:)B_\alpha=\left[a_1(\alpha)\ ,\ a_2(\alpha)\right](:)\left[b_1(\alpha)\ ,\ b_2(\alpha)\right]$$

$$A_\alpha(:)B_\alpha=\left[\frac{a_1(\alpha)}{b_2(\alpha)}\ ,\ \frac{a_2(\alpha)}{b_1(\alpha)}\right]\ ,\quad b_1(\alpha)>0\ ,\quad \forall\ \alpha\in[0\ ,\ 1]$$

It can be shown that the two methods for operating with fuzzy numbers are equivalent (Klir and Yuan, 1995).

1.3. Triangular fuzzy numbers

Triangular fuzzy number (TFN) is called to the continuous real fuzzy number, such that the shape of the membership function with the horizontal axis determines a triangle. Its membership function is linear, left and right, and reaches a value of one for a single real number (Kaufmann and Gil Aluja, 1990; Dubois and Prade, 1980).

Is determined by three real numbers $a_1 \leq a_2 \leq a_3$, it expression by α-cuts is:

$$A_\alpha = [(a_2 - a_1)\ \alpha\ + a_1\ ,\ (a_2 - a_3)\ \alpha\ + a_3] \qquad (1)$$

Usually represented by $\tilde{A} = (a_1, a_2, a_3)$ (Figure 1). For its simplicity, is used in many practical situations, especially when on a certain scale only three values are known: the minimum, maximum and the highest level of presumption (Kahraman, 2006, 2008).

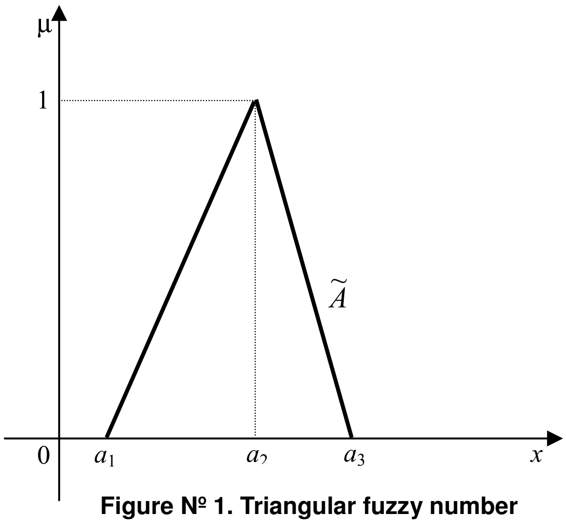
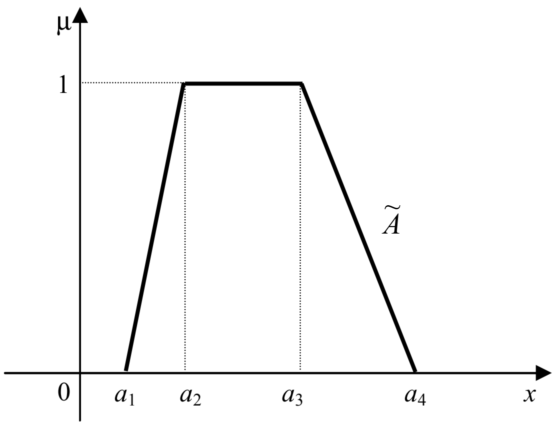


Figure Nº 1. Triangular fuzzy number
Source: Own Elaboration

1.4. Trapezoidal fuzzy numbers

Special Fuzzy Number (FNTr) is determined by four real numbers $a_1 \leq a_2 \leq a_3 \leq a_4$. Usually represented by $\tilde{A} = (a_1, a_2, a_3, a_4)$ (Figure 2); the α-cuts are (Kaufmann and Gil Aluja, 1990; Bojadziev and Bojadziev, 1997; Dubois and Prade, 1980):



(2)

Figure Nº 2. Trapezoidal fuzzy number
Source: Own Elaboration

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2. Fuzzy function or fuzzy number function

In this paper is considered fuzzy function at all correspondence between fuzzy numbers (Buckley et al., 2010; Grabisch, 2009), the function is also called fuzzy number (Kaufmann and Gupta, 1985). The expresion $F(\tilde{X})=\tilde{Y}$ denotes a fuzzy function of the independent variable $\tilde{X}$. Uslly, $\tilde{X}$ is a triangular or trapezoidal fuzzy number and $\tilde{Y}$, it is any other fuzzy number.

Fuzzy function can be obtained as extensions of real functions of real variable by using: a) the Extension Principle and b) the α-cuts of a fuzzy number $\tilde{X}$.

a) Given $f:[a,b]\rightarrow\Re$ / $f(x)=y$ it can be extended to $F(\tilde{X})=\tilde{Y}$ in the following way:

$$\{\mu_{\tilde{X}}(x)/f(x)=y,\ a\leq x\leq b\} \quad (3)$$

The ecuation (3) define the membership function of $\tilde{Y}$ for any fuzzy number $\tilde{X}$ in $[a,b]$.

b) Given $f:[a,b]\rightarrow\Re/¿$ ¿ $f(x)=y$, it express $\tilde{X}\subset[a,b]$ for the α-cuts and the fuzzy extent is obtained $F(X_\alpha)=Y_\alpha,\ \forall\ \alpha\in[0,1]$.

In this paper the fuzzy functions are obtained as extensions of real functions of real variable using the α-cuts of fuzzy number $\tilde{X}$.

2.1. Domain of regularity

If $\Re(\tilde{X})$ is the set of all fuzzy numbers ℜ. Given the fuzzy number $\tilde{X}\in\Re(\tilde{X})$ express by their α-cuts $X_\alpha=[x_1(\alpha),\ x_2(\alpha)]$ and the function $f: D\rightarrow\Re$ monotonically increasing in this interval, the fuzzy extent $F(\tilde{X})=\tilde{Y}$ is regular in the domain $D\subseteq\Re$ if and only if (Figure 3):

$\forall\ \alpha\in[0,1]$, $[x_1(\alpha);\ x_2(\alpha)]\subseteq D$:

$$F(X_\alpha)=F[x_1(\alpha),\ x_2(\alpha)]$$

$$F(X_\alpha)=[f(x_1(\alpha)),\ f(x_2(\alpha))] \quad (4)$$

The set in which a fuzzy function is called domain regulating regularity (Lazzari et al., 1998).

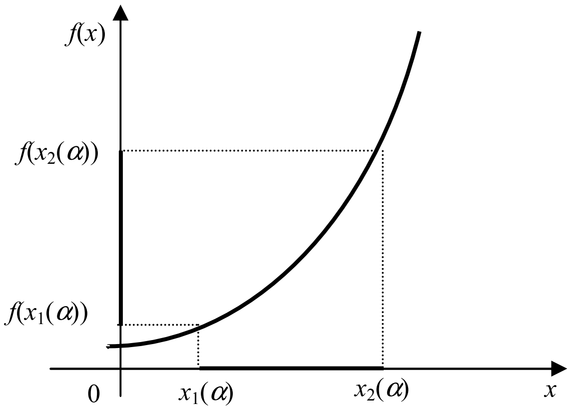

Figure No. 3: Domain regularly
Source: Own Elaboration, based on Kaufmann and Gupta (1985)

Given the fuzzy number $\tilde{X} \in \Re(\tilde{X})$ which α-cuts are $X_\alpha = [x_1(\alpha), x_2(\alpha)]$ and a function $f: D^* \to \Re$ monotone decreasing in said range, the fuzzy extent $F(\tilde{X}) = \tilde{Y}$ is not regular in the domain $D^* \subseteq \Re$ if and only if (Figure 4):

$\forall \alpha \in [0, 1]$, $[x_1(\alpha); x_2(\alpha)] \subseteq D^*$

$$F(X_\alpha) = F[x_1(\alpha), x_2(\alpha)]$$

$$F(X_\alpha) = [f(x_2(\alpha)), f(x_1(\alpha))] \tag{5}$$

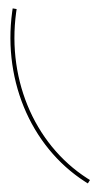

Figure Nº4. Dominion of not regularly
Source: Own Elaboration, based on Kaufmann and Gupta (1985)

The set in which a fuzzy function is not regular is called a non regular domain (Lazzari et al., 1998).

In cases where the domain is irregular, to obtein the fuzzy extension of $f(x) = y$ it will have to partition it. Figure 5 shows that $[x_1(\alpha); x_2(\alpha)]$ is an irregular domain and it can be divided into $[x_3(\alpha); x_2(\alpha)]$, which is a domain of regularity and $[x_1(\alpha); x_3(\alpha)]$ non regularity.

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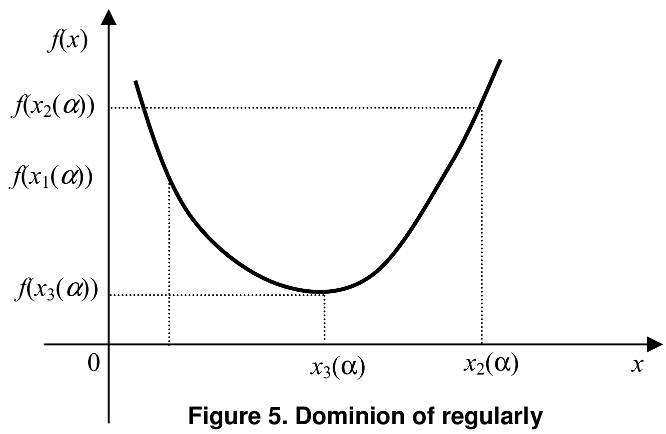

Figure 5. Dominion of regularly
Source: Own Elaboration based on Kaufmann and Gupta (1985)

2.2. Derivative of a fuzzy function

Being $y = f(x)$ a non-negative function, monotonic increasing, continuous and concave positive $[a;b] \subset \Re^+$, with non-negative derivative $y' = f'(x)$ in that interval. Their fuzzy expressions, $\tilde{Y} = F(\tilde{X})$ and $F'(\tilde{X})$, will be regular functions on the interval $[a;b] \subset \Re^+$ (Kaufmann and Gupta, 1985). For those functions that do not meet these conditions should be analyzed case by case basis.

Are considered the α-cuts of $F(\tilde{X})$ and of $F'(\tilde{X})$ for $X_\alpha = [x_1(\alpha), x_2(\alpha)]$, and (4) its apply: $\forall \alpha \in [0,1]$: $F(X_\alpha) = [f(x_1(\alpha)), f(x_2(\alpha))]$ and $F'(X_\alpha) = [f'(x_1(\alpha)), f'(x_2(\alpha))]$ where $[x_1(\alpha), x_2(\alpha)] \subset [a,b] \subset \Re^+$.

As $\tilde{X}$ it's a FN $F(\tilde{X})$ and $F'(\tilde{X})$ are also fuzzy numbers, and their membership functions are convex and normal (Lazzari et al., 1995, 1998).

For example: given the function $f: \Re \to \Re \,/\, f(x) = x^2 + x$, non-negative, continuous and monotone increasing in $\Re^+$, its derivative is $f: \Re \to \Re \,/\, f'(x) = 2x + 1$.

Their fuzzy extensions for fuzzy number are $\tilde{X} \subset \Re^+$

$$F(\tilde{X}) = \tilde{X}^2 (+) \tilde{X} \quad \text{and} \quad F'(\tilde{X}) = 2\tilde{X} (+) 1$$

Are regular in the domain $D = \Re^+$, si $X_\alpha = [x_1(\alpha); x_2(\alpha)] \subset \Re^+$

$$F(X_\alpha) = [x_1^2(\alpha) + x_1(\alpha), x_2^2(\alpha) + x_2(\alpha)] \tag{6}$$

$$F'(X_\alpha) = [2x_1(\alpha) + 1, 2x_2(\alpha) + 1] \tag{7}$$

Given the TFN $\tilde{X} = (1,3,4)$, the function (1) is apply to obtain their α-cuts: $X_\alpha = [2\alpha + 1, -\alpha + 4]$.

For (6) $$F(X_\alpha) = [4\alpha^2 + 6\alpha + 2, \alpha^2 - 9\alpha + 20] \tag{8}$$

For (7) $$F'(X_\alpha) = [4\alpha + 3, -2\alpha + 9] \tag{9}$$

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$$F'(\tilde{X}) = (3, 7, 9)$$

As shown in (8) the $F(\tilde{X})$ is not a TFN, while $F'(\tilde{X})$ yes it is as seen in (9) (Figure 6).

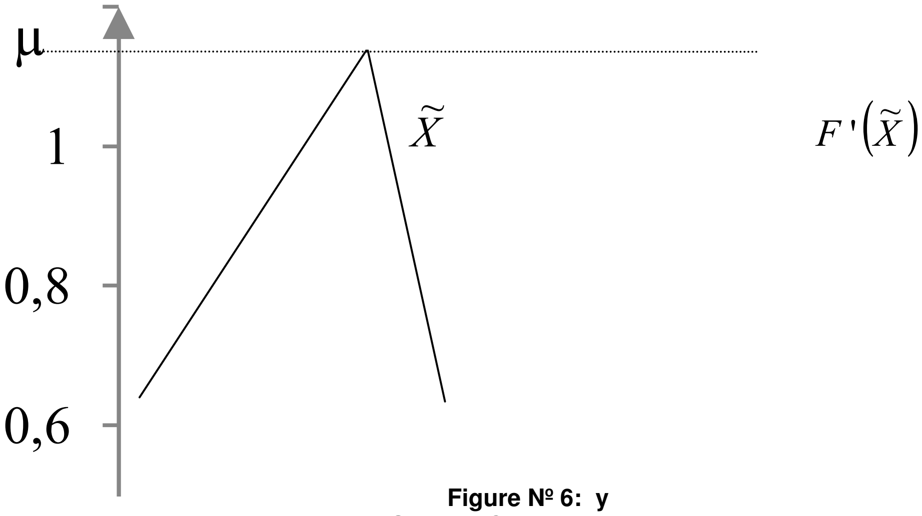

Figure Nº 6: y
Source: Own Elaboration

3. Study case

A mathematical model can accurately describe the data of a problem or provide an acceptable approximation of it.

The fuzzy set theory provides a valuable framework for the representation of this uncertainty in decision-making (Yager, 1996). Fuzzy systems have the ability to model forms of reasoning not necessary, that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision.

Economics is often observed that relations between magnitudes and, therefore, the functions that are expressed indeterminately or some of its parameters can take values imprecise or vague. In an uncertain environment, the decision maker may want to reach certain levels of aspiration that can not be defined neatly, as well as the variables considered can support small detours. Consequently, different elements of the functions that can be associated fuzzy and blurring can be expressed in different ways.

The address of a company must maintain a register and control operating costs, income from the sale of their products or services as well as the benefits. Three functions provide a measure of these quantities: total cost function, total revenue function and profit function (Tan, 2002).

Being x the number of units produced or sold in a product.

$C(x)$ total cost of manufacturing x units of the product.

$I(x)$ total revenue from the sale of x units of the product.

$B(x)$ total profit for the marketing of x products.

Not always in a production process can pinpoint the number of units to be produced and to be sold in a given period of time. Business experts may estimate a value below which no reference amount will be located, another above which not the same and a figure considered as possible will be found. This approach can be represented by a TFN.

Under these circumstances, the above functions can be extended to fuzzy field.

$C(\tilde{X})$ for $\tilde{X}=(x_1, x_2, x_3)$ indicates the lowest total cost to produce x_1 production units; higher costs, which correspond to the production of x_3 units; and cost with higher level of presumption, if x_2 items are produced.

$I(\tilde{X})$ indicates the lowest total income, which corresponds to the sale of x_1 units of product; the highest total income, correspond to sell x_3 units; and the most probable income is, if its sell x_2 units. Finally, $B(\tilde{X})$ shows the benefit of at least marketing x_1 and as maximum x_3 units.

The total cost function of a manufacturer is given by

$$C(x)=0.3\,x^2+2\,x+850 \tag{10}$$

Where C provides the total cost of producing x units of product.

Given the market uncertainty, the entrepreneur estimates that in the next period will need to produce at least 240 units, no more than 330 and possible 300 units of product. Under this assumption, we can express the quantity produced by the TFN $\tilde{X}=(240,\ 300,\ 330)$, which α-cuts are:

$$X_\alpha=[60\,\alpha+240\ ,\ -30\alpha+330\ ] \tag{11}$$

As the fuzzy function $C(\tilde{X})$ is regular in $\Re^+$, it calculate $C(\tilde{X})=0.3\,\tilde{X}^2+2\,\tilde{X}+850$ as the extension of the cost function given in (10), by using the α-cuts of $\tilde{X}$ and (4).

$$C(X_\alpha)=[1080\,\alpha^2+8760\,\alpha+18610\ ,\ 270\,\alpha^2-6000\,\alpha+34180\ ] \tag{12}$$

It is noted that the expression (12) represents a non-triangular fuzzy number (Figure 7).

In (12) if $\alpha=0$: $C(X_0)=[18610\ ,\ 34180\ ]$ and if $\alpha=1$: $C(X_1)=[28450\ ,\ 28450\ ]$.

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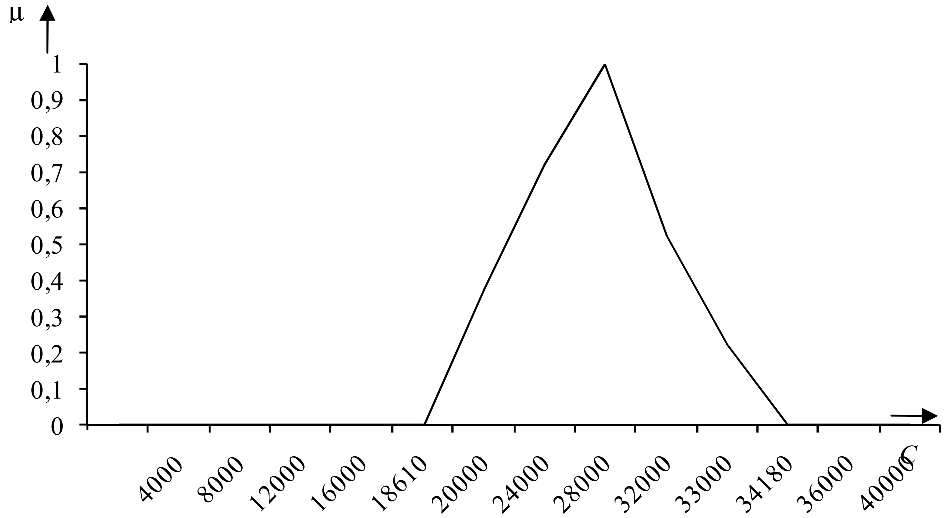

Figure Nº 7: Total cost blurry
Source: Own Elaboration

If the results for the present case are observed, we can say that the total cost will not be less than $ 18,610, or more than $ 34,180 and its value will be $ 28,450.

The manufacturer assumes that the units produced can be sold at a price of 1000 pesos. Its total revenue function is given by (13).

$$I(x) = 1000\ x \qquad (13)$$

Besides, its estimates that next sales period will not be less than 240 units, shall not exceed 330 and the chances are that 300 units of the product sold. This information by the triangular fuzzy number is expressed $\tilde{X} = (240\ ,\ 300\ ,\ 330\ )$. As the fuzzy number function $\tilde{X}$ is regular in $\Re^{+}$, is calculate $I(\tilde{X}) = 1000\ \ \tilde{X}$, in accordance with (4).

$$I(X_{\alpha}) = [6000\,\alpha + 240000\ ,\ -30000\,\alpha + 330000\ ] \qquad (14)$$

The fuzzy income is represented by the TFN $\tilde{I} = (240000,\ 300000,\ 330000)$, which can be observed in Figure 8.

This result indicates that the expected income shall not be less than 240000 or monetary units will exceed 330,000, and it is most likely to be 300,000 units.

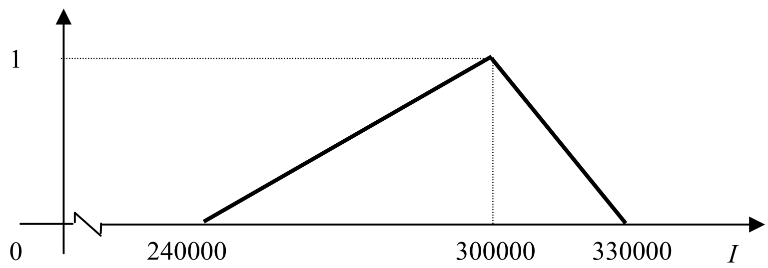

Figure Nº8: Fuzzy income
Source: Own Elaboration

The profit function can be obtained as the difference between the roles of income and total costs.

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$$B(x)=I(x)-C(x)$$

If we rest (10) from (13) we obtain:

$$B(x)=-0.3x^2+998x-850 \quad (15)$$

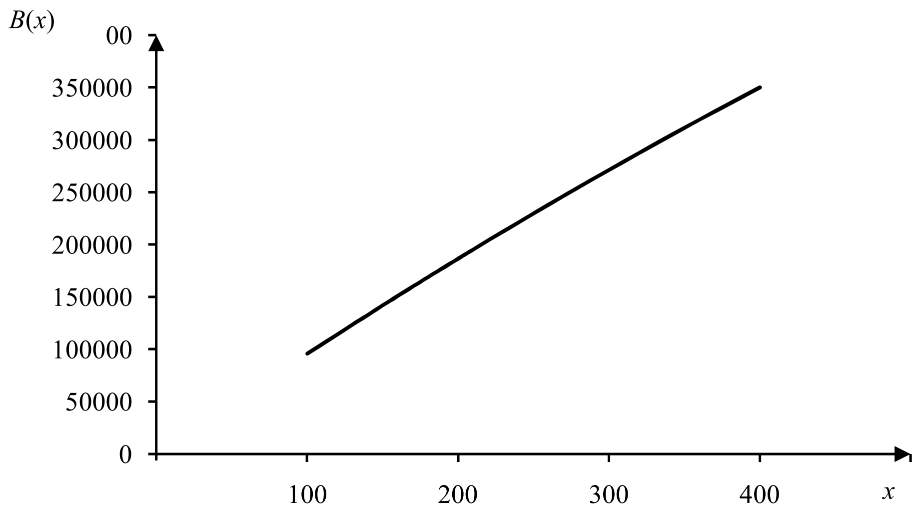

Figure Nº 9: Function benefit on the range [100,400]
Source: Own Elaboration

As its shows in Figure 9, the function $B(x)$ is monotone increasing in the interval $[240, 330] \subset \Re^+$, its fuzzy extension $B(\tilde{X})$ is regular in this interval for $\tilde{X}=(240, 300, 330)$ and is obtained by applying (4).

$$B(X_\alpha)=[-1080\alpha^2+51240\alpha+221390 ,\ -270\alpha^2-24000\alpha+295820] \quad (16)$$

In (16) if $\alpha=0$: $B(X_0)=[221390, 295820]$ and if $\alpha=1$: $B(X_1)=[271550, 271550]$.

If all production is sold, the values of $B(X_0)$ indicate the benefit to 240 and 330 units sold the product and $B(X_1)$ shows the benefit for 300.

This information is valuable to the employer who shall promote actions that lead to achieving the next most optimistic scenario values.

Among the applications that have the derivative of a function is getting called marginal rates, which can explore the reasons for change involving financial aggregates (Hoffmann et al., 2004).

The marginal cost measures the rate at which the cost increases with respect to the variation of the amount produced and can be calculated, approximately, using the derivative function of the cost to the level of production considered (Harshbarger and Reynolds, 2005).

Being the function cost $C(x)$ given in (10), its derivative is calculated: $C'(x)=0.6x+2$ and its obtain the fuzzy extension $C'(\tilde{X})=0.6\tilde{X}+2$, which is regular in $[240, 330] \subset \Re^+$.

$$C'(X_\alpha)=[36\alpha+146 ,\ -18\alpha+200] \quad (17)$$

if $\alpha=0$: $C'(X_0)=[146, 200]$ and if $\alpha=1$: $C'(X_1)=[182, 182]$

Therefore the marginal fuzzy cost is given by the TFN $C'(\tilde{X})=(146\ ,182\ ,200)$, which indicates the marginal increase in costs to take place no less than 240, no more than 330 and as much as possible, 300 units (Figure 10).

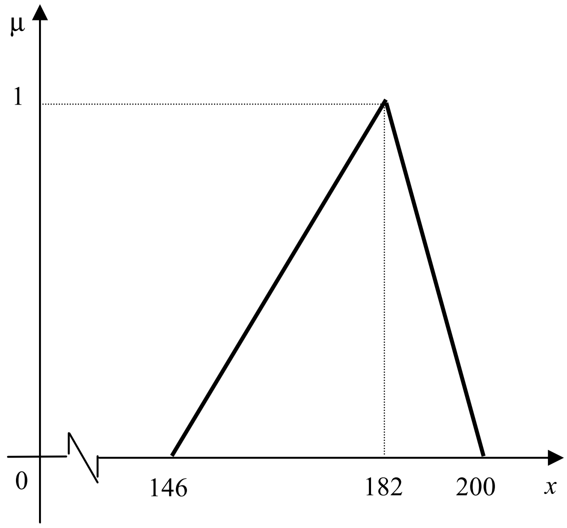

Figure Nº 10: Marginal cost increase
Source: Own Elaboration

To calculate approximate total cost for a minimal fuzzy production increase is necessary to add $C(\tilde{X})$ and $C'(\tilde{X})$ that means (12) y (17).

$$C(x_\alpha)(+)C'(x_\alpha)=[1080\ \alpha^2+8796\ \alpha+18756\ ,\ 270\ \alpha^2-6018\ \alpha+34380] \quad (18)$$

The expression (18) is not a TFN. If $\alpha=0$: $C(X_0)(+)C'(X_0)=[18756\ ,34380]$ and if $\alpha=1$: $C(X_1)(+)C'(X_1)=[28632\ ,28632]$.

This result indicates that if a minimal increase in production is made, the total cost will not be less than $ 18,756 or more than $ 34,380.

CONCLUSION

As discussed in this paper, under uncertainty, the functions of fuzzy number is an appropriate tool to represent the functions of the total cost, total revenue and profit, given that provide insight into the different scenarios when making decisions about number of units of a good to produce in the next period. The information provided allows you to see clearly what the real business prospects are in terms of gain that can be achieved with the different levels of production and sales.

According to available information, quantities produced and sold are also represented by a trapezoidal fuzzy number $\tilde{X}=(x_1,x_2,x_3,x_4)$, where x_1 indicates a value below which no reference amount is located, x_4 expresses the value above which is not the same and the

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possible values belong to the interval. $[x_2 , x_3]$.

The fuzzy functions can be used to study other economic functions such as supply, demand, consumption and production. The use of marginal rates is wide in business and economics; plus the marginal cost, calculated in this work is useful to obtain, among others, marginal revenue, marginal benefit, marginal productivity and marginal tendencies to consume and save; and uncertainty can be incorporated in each case, by means of fuzzy numbers.

The use of fuzzy methodology for decision making in issues management and economics allows for more efficient use of resources and provides more information to the decision maker when rigid mathematical techniques are applied.

Implement flexible models employing fuzzy methodologies to pose and solve problems of management and economics is useful to identify and understand the difficulties that arise in their analysis, and to generate new scenarios of reflection when choosing means and score goals.

BIBLIOGRAPHY

Please refer to articles Spanish Bibliography.

BIOGRAPHICAL ABSTRACT

Please refer to articles Spanish Biographical abstract.